Warm-up

Simplify.

1) a) $\dfrac{b^4 \cdot b^7}{b^6}$

b) $\left(\dfrac{3x^2}{4x}\right)^{-1}$

c) $(s^4t^7)^3$

d) $\dfrac{x^2y^5}{(x^2)^4}$

2) Jen put $450 in an account with 5% interest compounded monthly for years. What will her balance be?

3) A city population is decreasing by 6% per yea If the current population i 45,000, what will it be in 1 years?

Warm-up

Simplify.

1) a) $\dfrac{b^4 \cdot b^7}{b^6} \rightarrow \dfrac{b^{11}}{b^6} \rightarrow \boxed{b^5}$

b) $\left(\dfrac{3x^2}{4x}\right)^{-1} \rightarrow \dfrac{4x}{3x^2} \rightarrow \boxed{\dfrac{4}{3x}}$

c) $(s^4t^7)^3 \rightarrow \boxed{s^{12}t^{21}}$

d) $\dfrac{x^2y^5}{(x^2)^4} \rightarrow \dfrac{x^2y^5}{x^8} \rightarrow \boxed{\dfrac{y^5}{x^6}}$

Warm-up

2) Jen put $450 in an account with 5% interest compounded monthly for 6 years. What will her balance be?

$A = P\left(1+\frac{r}{n}\right)^{nt}$ $A = 450\left(1+\frac{0.05}{12}\right)^{12\cdot 6}$

$\boxed{\$607.0}$

3) A city population is decreasing by 6% per year. If the current population is 45,000, what will it be in 12 years?

$y = A(1-r)^{t}$ $y = 45000(1-0.06)^{12}$

$\boxed{21416 \text{ people}}$

1. What is the difference between simple and compound interest?

Compound interest makes interest on interest and simple interest does not.

2. Which will yield the larger balance in one year?

a. $3000 at 6% annual interest, compounded twice a year **$3,182.70**

b. $3000 at 6% annual interest, compounded four times a year

$3,184.09

3. $2000 is deposited in an account that pays 8% annual interest, compounded monthly. What is the balance in five years? **$2,979.69**

4. $2000 is deposited in an account that pays 8% annual interest, compounded annually. What is the balance in five years? **$2,938.66**

5. Is $y = 5\left(\frac{6}{5}\right)^x$ a model for exponential growth or decay? Explain.

Growth 6/5 > 1

6. Is $y = \frac{1}{4}\left(\frac{3}{4}\right)^x$ a model for exponential growth or decay? Explain.

Decay 3/4 < 1

7. $800 is deposited in an account that pays 9% annual interest, compounded ~~annually~~. weekly

Find the balance after four years.

~~$1,129.27~~

$1146.31

8. $100 is deposited in an account that pays 6% annual interest, compounded annually. Find the balance after four years. $126.25

9. $2250 is deposited in an account that pays 6% annual interest, compounded quarterly. Find the balance after ten years. $4081.54

10. $1800 is deposited in an account that pays 5% annual interest, compounded monthly. Find the balance after one year. $1,892.09

11. How much must you deposit in an account that pays 6.25% interest, compounded annually, to have a balance of $700 after two years?

$$700 = P(1 + 0.0625/365)^{2*365}$$

$$P = \$617.75$$

12. How much must you deposit in an account that pays 8% interest, compounded monthly, to have a balance of $1000 after one year?

$$1000 = P(1 + 0.08/12)^{12}$$

$$P = \$923.36$$

13. You have inherited an emerald ring that has an appraised value of $2400 in 1960. It is now 1996, and the appraised value of the ring has increased by approximately 6% each year. What is the value now?

$$A = 2400(1 + 0.06)^{36}$$

$$A = \$19,553.40$$

14. Vincent Van Gogh's painting "Irises" was auctioned for 53.9 million dollars in 1987. Suppose that it sold for $50 in 1889, when it was painted. Assuming exponential growth, by what percent did its value increase each year?

$$53,900,000 = 50(1 + r)^{98}$$

$$r = 15.23\%$$

Unit 8: Exponential & Logarithms

Continuous Interest, Present & Future Value

Growth
Decay
Compound Interest

3 Equations Last Class

$$y = A(1 + r)^x$$

$$y = A(1 - r)^x$$

$$A = P(1 + \frac{r}{n})^{nt}$$

3 More Equations Today

Continuous Interest

Present Value

Future Value

<u>Base *e* and Natural Logarithms</u>

Suppose you deposited \$1 in the bank for 1 year at 100%

Round to 6 decimal places.

n Number of times compounded	$A = P(1 + r/n)^{nt}$	A
1 (yearly)	$A = 1(1 + 1/1)^{1(1)}$	2
12 (monthly)	$A = 1(1 + 1/12)^{12(1)}$	2.613035
52 (weekly)	$A = 1(1 + 1/52)^{52(1)}$	2.692597
365 (daily)		2.714567
8760 (hourly)		2.718127
525600 (every minute)		2.718279
31536000 (every second)		2.718282

Continuously Compounding Interest Equation

$$A = Pe^{(rt)}$$

A = Account Balance *(The final amount)*

P = Principle *(The initial amount)*

r = Annual interest rate

t = time in years

e = Natural Number

2^{nd} ÷

Exponential Word Problems

1) Look for continuous

2) Look for compounded

3) Check vocab. for growth / decay

4) Sub into the appropriate equation

5) Solve

Compounded Continuously Problems

Example 5 You deposit \$10,000 in an account that pays 6% interest. Find the balance after 10 years if the interest is compounded.

$$A = P(1+\frac{r}{n})^{nt}$$

a. quarterly :

$A = 10{,}000(1 + .06/4)^{(10\cdot 4)}$

$A = \$18{,}140.18$

b. continuously : $A = Pe^{rt}$

$A = 10000e^{(0.06\cdot 10)}$

$A = \$18{,}221.19$

Example 6 \$600 is deposited in an account that pays 7% annual interest, compounded continuously. What is the balance after 8 years ?

$A = 600e^{(0.07\cdot 8)}$

$A = \$1050.40$

Example 7 $1250 is deposited in an account that pays 6.5 % annual interest, compounded continuously. What is the balance after 8 years ?

$$A = 1250e^{(0.065 \cdot 8)}$$
$$A = \$2102.53$$

Example 8

$3000 is deposited in an account that pays 5% annual interest. Compare the balance at the end of 10 years for continuous compounding of interest with the balance for quarterly compounding.

a) continuously:

$$A = Pe^{rt}$$
$$A = 3000e^{(0.05 \cdot 10)}$$
$$A = \$4946.16$$

b) quarterly:

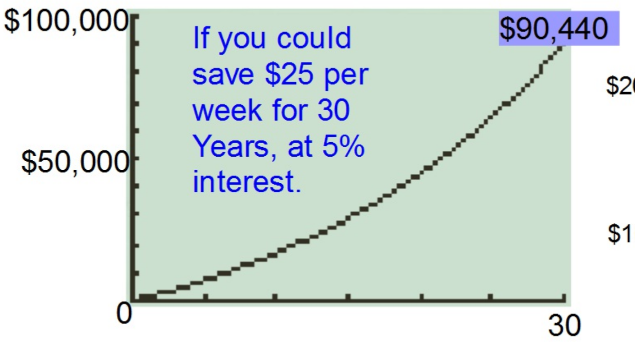
$$A = P(1 + \frac{r}{n})^{nt}$$
$$A = 3000(1 + 0.05/4)^{(4 \cdot 10)}$$
$$A = \$4930.86$$

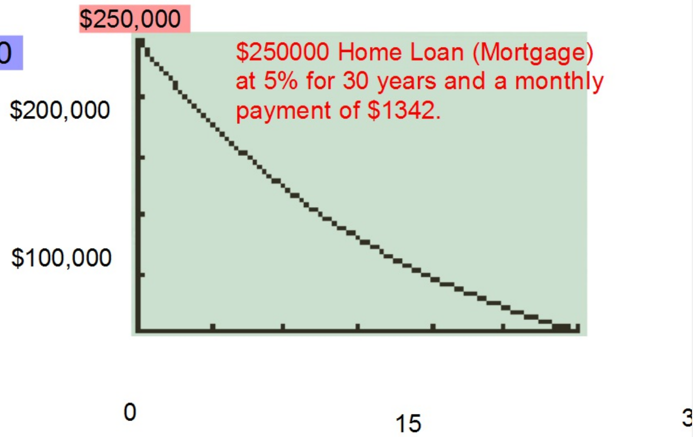
Annuities

S — Savings or loans that have a constant
L — payment, made at regular intervals.

Savings - for a Saving's Club account the same amount is deposited each week or month.

Loan - for a car or house the payment is always the same and paid monthly.





Loan Annuities

Present Value Function

$$P_n = p\left[\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}\right]$$

P_n = amount borrowed
P = amount of each payment
r = interest rate as a decimal
n = number of payments per year
t = total number of years

Most Questions

Some

Saving Annuities
Future Value Function

$$F_n = p\left[\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}\right]$$

F_n = amount in the account
p = amount of regular deposit
r = interest rate as a decimal
n = number of payments per year
t = total number of years

F_n → Most

p → Some

$$P_n = P\left[\frac{1-(1+i)^{-n}}{i}\right]$$, where i is the interest rate for the period. $\left(i = \frac{r}{n},\ n = nt\right)$

a. Use the present value formula to find the monthly payment you would pay o home mortgage if the present value is $121,000, the annual interest rate is 7. and payments will be made for 30 years.

$$121000 = P\left[\frac{\left(1-\left(1+\frac{0.075}{12}\right)^{-12\cdot 30}\right)}{\left(\frac{0.075}{12}\right)}\right]$$

P = $846.05

payments of P dollars for n periods. The formula for the present value is

$$P_n = P\left[\frac{1-(1+i)^{-n}}{i}\right]$$, where i is the interest rate for the period. ($i = \frac{r}{n}$, n = nt)

b. How much is the monthly payment if the borrowers choose a loan with a 20-term and an interest rate of 7.25%?

$$121000 = P\left[\frac{1-\left(1+\frac{0.0725}{12}\right)^{-12\cdot 20}}{\frac{0.0725}{12}}\right]$$

P = $956.35

c. How much will be paid in interest over the life of each mortgag

A $846.05 \cdot 12 \cdot 30 = 304578$ Total Paid

-121000

$\$183578$ Interest

B $956.35 \cdot 12 \cdot 20 =$

$\$229524$

-121000

$\$108524$

d. Explain why a borrower might choose each mortga

B → Lower total cost

A → More manageable monthly payment

2) Tom is planning to buy an $18,000 car. The loan is for 5 years at a rate of 10 ½ %.

$$18000 = P\left[\frac{1-\left(1+\frac{0.105}{12}\right)^{-12\cdot 5}}{\frac{0.105}{12}}\right]$$

a. What will be the monthly payments?

$386.89

b. How much money will he have paid the loan company for the car?

$23,213.40

c. How much interest will he have paid over the 5 years?

$5,213.40