Warm-up

1) A bacteria spreads at an exponential ra
of 6% per month. If there are 50 samples t
begin with, how many samples will there b
in 8 months?

$y = A(1+r)^t$ $y = 50(1+0.06)^8$

79.69 samples

2) A dead trees decays by 9% per year. If
tree is 500 lbs, how much will remain in 6
years?

$y = A(1-r)^t$ $y = 500(1-0.09)$

283.93 lbs

Unit 8: Exponential Functions

Finance Applications

An ant farm has a population of 150 ants that is growing 2% per month. What will the population be in 3 months?

Solving exponentials:

$3^x = 27$

$3^x = 3^3$

$x = 3$

$2^{x-3} = 32$

$2^{x-3} = 2^5$

$x - 3 = 5$

$x = 8$

$\log_2 32 = x - 3$

$2^{4x} = 284$

$\log_2 284 = 4x$

$\frac{\log 284}{\log 2} = 4x$

Ans/4 = x

$x = 2.037$

But what about...

$3^x = 17$

Logarithm

$\log_3 17 = x$

Change of Base

$x = \frac{\log 17}{\log 3} = 2.579$

Logarithms: the Other Side of Exponentials

But what about...

$3^x = 17$

Rewriting Logs and Exp.

base $^{\text{exponent}}$ = amount

$5^x = 408$ $\qquad$ $\log_3 x = 4$

$\log_5 408 = x$ $\qquad$ $3^4 = x$

log $_{\text{base}}$ amount = exponent

Examples : Write each equation in logarithmic form.

a. $x^3 = 64$

$\log_x 64 = 3$

b. $3^x = \frac{1}{27}$

$\log_3 \frac{1}{27} = x$

c. $10^x = 1000$

$\log_{10} 1000 = x$

$\log 1000 = x$

d. $a^k = d$

$\log_a d = k$

Examples: Write each equation in exponential form.

a. $\log_{125} x = \frac{2}{3}$

$125^{\frac{2}{3}} = x$

b. $\log_x 2 = \frac{1}{3}$

$x^{\frac{1}{3}} = 2$

c. $\log_6 x = 0$

$6^0 = x$

d $\log_h z = c$

$h^c = z$

Examples: Evaluate each logarithmic expressions.

a. $\log_2 64$

$\frac{\log 64}{\log 2} = 6$

b. $\log_3 243$

$\frac{\log 243}{\log 3} = 5$

c. $\log_{256} 4$

$\frac{\log 4}{\log 256} = 0.25$ or $\frac{1}{4}$

Change of Base

$$\log_a b = \frac{\log b}{\log a}$$

Examples: Solve.

a) $3^{2n+1} = 81$

$\log_3 81 = 2n+1$

$4 = 2n+1$

$3 = 2n$

$n = 1.5$ or $\frac{3}{2}$

OR $3^{2n+1} = 3^4$

$2n+1 = 4$

Some other basics:

Common Log - logarithm with base 10

ex. $\log x = 3 \Rightarrow 10^3 = x$

Natural Log - logarithm with base e

ex. $\ln x = 4 \Rightarrow e^4 = x$

$\log_e 2 = \ln 2$

Steps to solve:

1) **Isolate the exponential piece**
2) **Rewrite as a logarithm**
3) **Evaluate Log (change of base)**
4) **Finish Solving if Needed**

$5 + 4^{2x-3} = 27$

$4^{2x-3} = 22$

$\log_4 22 = 2x - 3$

Ans $+ 3 = 2x$

Ans $/ 2 = x$

$x = 2.615$

$2e^{4x+7} - 8 = 32$

$2e^{4x+7} = 40$

$e^{4x+7} = 20$

$\ln 20 = 4x + 7$

Ans $- 7 = 4x$

Ans $/ 4 = x$

$x = -1.001$

$\log_e 20 = 4x + 7$

$3^{x-4} = 2^{x-3}$

Still rewrite as a log Just have to pick a side.

$\log_3 2^{x-3} = x-4$

$(x-3)\log_3 2 = x-4$

$x\log_3 2 - 3\log_3 2 = x-4$

$x\log_3 2 - x = -4 + 3\log_3 2$

$x(\log_3 2 - 1) = -4 + 3\log_3 2$

$$x = \frac{-4+3\log_3 2}{\log_3 2 - 1}$$

$x = 5.710$

Sometimes picking a side matters.

Pick the side with more in the exponent to be the base of the log.

$4^x = 3^{x+1}$

$\log_3 4^x = x+1$

$x\log_3 4 = x+1$

$x\log_3 4 - x = 1$

$x(\log_3 4 - 1) = 1$

$$x = \frac{1}{\log_3 4 - 1}$$

$x = 3.819$